

QUANTIFICATION OF ENERGY LOSSES IN REAL MECHANICAL SYSTEMS

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Abstract: Creation of an atmosphere that enables students to grasp the essence of phenomena and scientific concepts they are studying is closely associated with experience gained in laboratories. Students often encounter difficulties in understanding the concept of energy transformation and conservation and the friction phenomenon. The paper presents a new method, based on Euler's research, which enables quantification of „losses” of mechanical energy in real mechanical systems. Applying the principles of Euler's method, an apparatus has been developed, enabling the quantification of very small “energy losses” of the order of 0.01 [J].

Key Words: Experiment/Rolling friction/ Energy losses/Engineering education

1. INTRODUCTION

Encouraging students to “fall in love with their subjects and intrinsically adopt a deep learning approach” [1] is one of the greatest engineering education challenges. Creation of an atmosphere that enables students to grasp the essence of phenomena and scientific concepts they are studying is closely associated with experience gained in laboratories. A number of authors recognize the importance of hands-on learning and provide their students with the means and opportunities to learn how to analyse and solve complex engineering problems by engaging in experiments and by personally handling the devices, experimental sets and instruments [2].

One of the challenges in engineering education is related to the concept of energy transformation and conservation and the friction phenomenon. Students often encounter difficulties in understanding these phenomena [3-5]. Therefore, the analysis of forces and energy transformation and dissipation related to this phenomenon should include the demonstration of the connection between well-known theories and the results obtained through experiments. Authors from universities worldwide propose diverse solutions based on the inclined plane and the analysis of different aspects of the friction phenomenon [6-12]. An undriven laboratory cart rolling down the inclined plane was used to predict the coefficient of rolling friction [6]. Some experiments focus on the acceleration of a low-friction cart rolling up and down an inclined plane [7], while others use the principle of conservation of energy to predict the stopping distance of a friction block sliding down an inclined ramp and compare it with the measured stopping distance [8]. Mechanical energy conservation law was used as a quantitative approach to the problem of different accelerations of cylinders with different inertia rolling down the inclined wooden plate [9]. The inclined plane was used as a ramp for initiating the movement of a ball rolling off the edge of a table, in order to experimentally determine angular velocity before and after the ball leaves the table and to compare it with theoretical calculations [10]. An analysis of a height reached by the ball rolling down a track of rectangular cross section along two opposite slanted planes showed the differences between theoretical predictions based on the energy conservation principle and experimental results [11]. The coefficient of rolling friction between relatively hard objects is likely to be less than 0.001. In that case the rolling object normally accelerates down the incline. Contrarily, a ball with a high coefficient of rolling friction can roll to a stop when it rolls without slipping down an inclined plane [12].

The study of friction has a long history and it represents a good example of the interface between abstract and formal theories and the reality of daily experience [3]. In 1748 paper “On the Decrease in Friction Resistance” by Leonhard Euler the dry friction law was given decades before Coulomb, often considered the founder of friction science, published his main publications. Nevertheless, the Euler–Coulomb dry friction law entered the history as one of the most needed in engineering calculations [13]. Euler's 1750 paper “On the friction of solid bodies” [14] gives an analysis of his experiments based on the body sliding along an inclined plane. The plane inclines from the horizontal position until the body comes to the point of descending. Euler expressed the dry friction coefficient in terms of the time and his formula allows one to determine the friction coefficient experimentally. This paper presents a new method, based on Euler's research and developed for both didactic and research purposes. The method enables quantification of “losses of energy” in real mechanical systems. An apparatus has been developed, enabling the quantification of very small “energy loss” of the order of 0.01 [J]. The apparatus was introduced into the laboratory exercises program within the course “Experiment in Mechanical engineering” at the Faculty of Engineering, University of Kragujevac.

2. THEORETICAL ANALYSIS

The steel ball rolls along the path of the length s , from point A to point B (Figure 1).

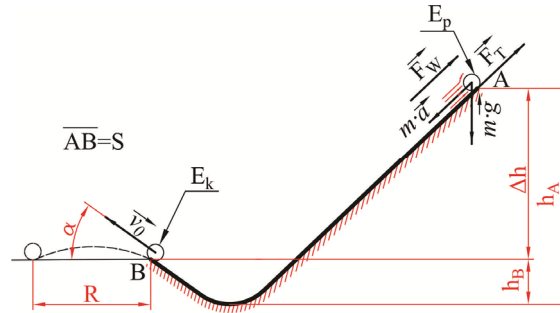


Figure 1. Schematic representation of the proposed method

The total energy of the ball at the initial position “A” is equal to its potential energy, which depends on the difference between heights h_A and h_B and on the mass of the ball (m). When the ball starts to roll, its potential energy (E_p) will transform into kinetic energy (E_k).

$$E_p = E_k + \Delta E \quad (1)$$

$$\Delta h = h_A - h_B \quad (2)$$

$$E_p = m \cdot g \cdot \Delta h \quad (3)$$

$$E_k = \frac{1}{2} \cdot m \cdot v^2 \quad (4)$$

At point “B”, the ball, with certain “energy losses” (ΔE), leaves the track of the length s at certain angle (α) and continues its motion according to the law of oblique projectile motion. The range of the projectile motion (R) is measured. “Energy losses” are equal to:

$$\Delta E = E_p - E_k = m \cdot g \cdot \Delta h - \frac{1}{2} \cdot m \cdot v^2 \quad (5)$$

The projectile velocity equals:

$$v^2 = \frac{R \cdot g}{\sin 2\alpha} \quad (6)$$

By substituting v^2 into equation (5), we get the final equation for calculation of the total “energy losses”:

$$\Delta E = m \cdot g \cdot \Delta h - \frac{R \cdot g \cdot m}{2 \sin 2\alpha} \quad (7)$$

Considering the small frontal area and the velocity of the ball, the air resistance force F_w can be neglected. Therefore, “energy losses” are considered to be caused by friction. The work done by friction (W_T) is the function of friction force $F_{T(s)}$ along the path s :

$$dW_T = F_{T(s)} \cdot ds, \quad (8)$$

$$W_T = \int_0^s F_{T(s)} \cdot ds \quad (9)$$

Considering that “energy loss” ΔE is equal to work done by friction W_T , ΔE can be calculated as follows:

$$\Delta E = W_T \equiv \bar{F}_{T(s)} \cdot s, \quad (10)$$

where $\bar{F}_{T(s)}$ is the average friction force along the path s .

3. EXPERIMENTAL RESEARCH

Measurements were performed using the apparatus (Figure 1a) containing two separate U profile (width: 12 mm, height: 8 mm) rolling tracks of different lengths.

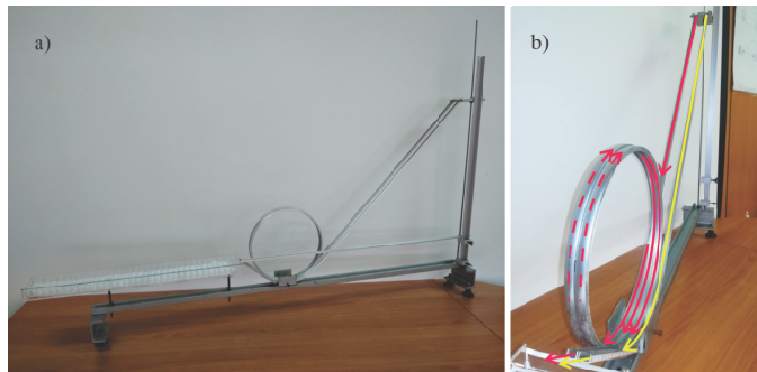


Figure 2. a) The apparatus; b) the longer path (red) and the shorter path (yellow)

The length of Track 1 (s_1) equals 3.02 [m], while the length of Track 2 (s_2) equals 1.195 [m] (Figure 1b). The longer track forms a double circle at some point, before straightening again. Both tracks are set at the identical initial height (h_A). They both end at the identical height (h_B), forming the identical angle α with the horizontal.

Measurements of the projectile range were performed using both tracks, 30 times each. Calculations were based on the equations presented within the theoretical analysis. The results are partially presented in Table 1.

Table 1 – The results of the experiments (partially presented)

No.	Track 1				Track 2			
	R_1 [mm]	E_{k1} [J]	ΔE_1 [J]	$\bar{F}_{T1}$ [N]	R_2 [mm]	E_{k2} [J]	ΔE_2 [J]	$\bar{F}_{T2}$ [N]
1	450	0.00475	0.00671	0.00222	530	0.00559	0.00586	0.00491
2	410	0.00433	0.00713	0.00236	530	0.00559	0.00586	0.00491
3	450	0.00475	0.00671	0.00222	550	0.00580	0.00565	0.00473
4	430	0.00454	0.00692	0.00229	530	0.00559	0.00586	0.00491
5	430	0.00454	0.00692	0.00229	430	0.00454	0.00692	0.00579
...	...	...	...	...	...	...	...	...
26	490	0.00517	0.00629	0.00208	490	0.00517	0.00629	0.00526
27	490	0.00517	0.00629	0.00208	490	0.00517	0.00629	0.00526
28	470	0.00496	0.00650	0.00215	570	0.00602	0.00544	0.00455
29	490	0.00517	0.00629	0.00208	550	0.00580	0.00565	0.00473
30	490	0.00517	0.00629	0.00208	570	0.00602	0.00544	0.00455
$\Delta h = 0.555$ [m]; $m = 2.10445$ [g]; $E_p = 0.01146$ [J]; $\alpha = 39^\circ$; $s_1 = 3.02$ [m]; $s_2 = 1.195$ [m]; Mean: $\bar{R}_1 = 462$ [mm]; $\Delta \bar{E}_1 = 0.00658$ [J]; $\bar{R}_2 = 534$ [mm]; $\Delta \bar{E}_2 = 0.00582$ [J]. Standard deviation: $SD_{R1} = 27.08862$ [mm]; $SD_{\Delta E1} = 0.00029$ [J]; $SD_{R2} = 42.14916$ [mm]; $SD_{\Delta E2} = 0.00044$ [J].								

4. DISCUSSION

When compared to the results of the experiments including the shorter track, the results related to the longer track show less average value of measured projectile ranges and, correspondingly, higher values of “energy losses” (Figures 3 and 4).

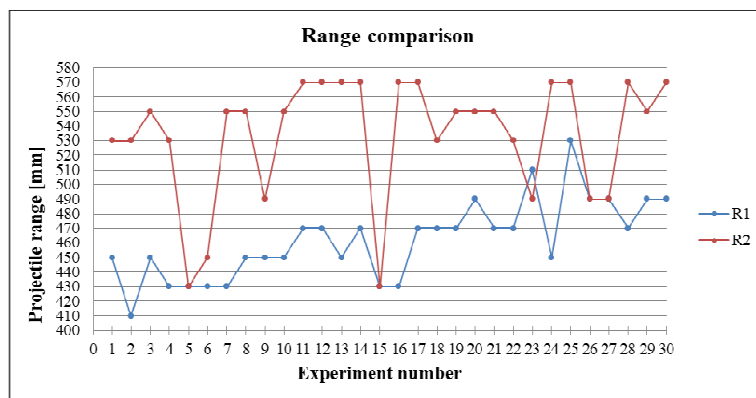


Figure 3. Measured projectile ranges on the longer track (R_1) and the shorter track (R_2)

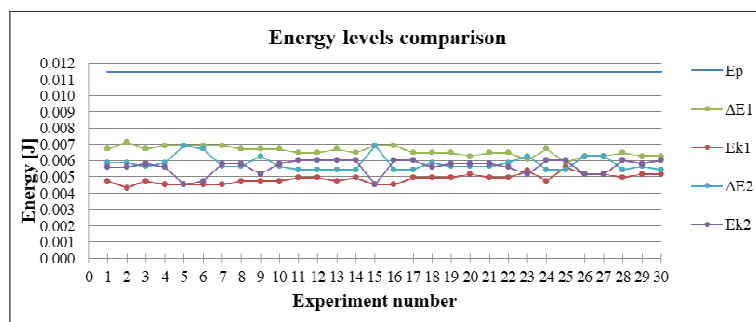


Figure 4. Kinetic energy levels and “energy losses” on Track 1 and Track 2, with respect to the potential energy at the starting position of the ball

Quantified energy levels are in the range of 0.004 to 0.007 [J] (Figure 4). "Energy losses" vary from 0.00544 to 0.00692 [J] on the longer track; on the shorter track "losses" are in range from 0.00586 to 0.00713 [J]. When compared to the results of the experiments which include the shorter track, the results related to the longer track show less average value of measured projectile ranges and, correspondingly, higher average value of "energy losses". However, when the projectile range R is identical for both tracks, i.e. the kinetic energy and "losses" are identical, average friction force along the shorter path is higher, which points out the combined effect of the path length and the friction force on the "loss" of energy. More detailed analysis would also consider the effect of surface condition on the results.

5. CONCLUSION

The method based on Euler's research enables the quantification of very small „losses" of mechanical energy in real mechanical systems and helps students to grasp the essence of the concept of energy transformation and conservation and the friction phenomenon. The apparatus was introduced into the redesigned laboratory exercises program within the course "Experiment in Mechanical engineering" at the Faculty of Engineering (FE), University of Kragujevac, along with a number of other low-cost "portable" devices, developed at the Faculty. A survey among 97 students was conducted in accordance with Blum's taxonomy with the aim to determine the effects of the work on new devices in small group class setting. The results [2] indicated that students perceived the work in small groups and the opportunity to personally conduct the experiment as a positive experience, which helped them to better understand theoretical framework, improve their motoric and communication skills. Experiments based on the method proposed in this paper allow teachers to initiate a discussion with students on various aspects of friction phenomenon and to perform further analyses and calculations.

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